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HEAT TRANSFER IN DEVELOPING FLOW OF VISCOPLASTIC MATERIALS THROUGH ANNULAR SPACES

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Abstract. *Heat transfer in the entrance-region laminar flow of viscoplastic materials inside annular spaces is analyzed, aiming at applications related to the drilling process of petroleum wells. The material is assumed to behave as a Generalized Newtonian Liquid, with a Herschel-Bulkley viscosity function. The governing equations are solved numerically via a finite volume method. Two boundary conditions were analyzed: constant wall heat fluxes and constant wall temperatures. The effect of yield stress and power-law exponent on the Nusselt number is investigated. It is shown that the entrance length decreases as the material behavior departs from Newtonian.*

Keywords: *Annular flows, Viscoplastic materials, Drill muds.*

1. INTRODUCTION

The present work analyzes the developing flow and heat transfer of viscoplastic materials through the annular spaces between two coaxial tubes. Two different thermal boundary conditions at the inner wall are investigated: uniform wall heat flux and uniform temperature. The outer wall was considered adiabatic. The governing conservation equations are solved numerically via a finite volume method. The material is assumed to behave as a Generalized Newtonian Liquid (GNL), with a Herschel-Bulkley viscosity function (Bird *et al.*, 1987) in order to model the viscoplastic behavior (i.e., non-zero yield stress) of the flowing material.

Many industries deal with viscoplastic materials in their processes. An important example is the drilling process of petroleum wells, where the drill mud is injected through the well. These materials should have certain rheological properties to ensure the success of a drilling operation. They should have the correct density to provide the pressure needed for well integrity and for avoiding premature production of hydrocarbons. Also, to drag the rock chips generated by the drill with reasonably low pumping power, a shear-thinning rheological behavior is highly convenient. In order to evaluate the correct rheological properties of these kind of materials, which are strong functions of temperature, the temperature field throughout the flow must be known.

Several papers discuss heat transfer of non-Newtonian fluids in axisymmetric flows. Bird *et al.* (1987), Irvine and Karni (1987), Joshi and Bergles (1980a, 1980b), Scirocco *et al.* (1985) and analyzed the heat transfer in flows of Power-Law fluids through tubes and proposed correlations for the Nusselt number. Vradis *et al.* (1992) analyzed numerically the heat transfer problem for developing flows of Bingham materials with constant properties through tubes. They considered the case of simultaneous velocity and temperature development. Nouar *et al.* (1994) presented an experimental and theoretical heat transfer study for Herschel-Bulkley materials inside tubes. They considered fully developed velocity profiles and investigated the entrance region. The impact of temperature-dependent rheological properties on velocity profiles and Nusselt numbers was also discussed. In a more recent work, Nouar *et al.* (1995) analyzed numerically the heat transfer to Herschel-Bulkley materials flowing through tubes. They considered temperature-dependent consistency index and simultaneous velocity and temperature development, and neglected axial diffusion of heat. Some correlations for local Nusselt number and pressure gradient were proposed. Soares *et al.* (1999) studied the developing flow of Herschel-Bulkley materials inside tubes, for constant and temperature-dependent properties, taking axial diffusion into account. Among other results, they observed that the temperature-dependent properties do not affect qualitatively pressure drop or the Nusselt number. Also, it was shown that axial diffusion is important near the tube inlet.

Heat transfer to viscoplastic materials through annular flows was investigated experimentally by Naimi *et al.* (1990). In their work, the inner cylinder was able to rotate, and a secondary flows appears due to rotation. The heat transfer coefficient was obtained as a function of the axial coordinate and angular velocity of the inner cylinder. More recently, Soares *et al.* (1998) studied heat transfer in a fully developed flow of Herschel-Bulkley materials through annular spaces, with insulated outer walls and with uniform heat fluxes at the inner walls. For that case it was observed that the Nusselt number depends quite little on the rheological properties.

2. ANALYSIS

The flow under study is steady and axisymmetric. The outer and inner radii are R_0 and R_i, respectively. The thermophysical and rheological properties of the flowing material are considered to be independent of temperature as a first approximation. The governing conservation equations are solved numerically via a finite volume method, to be described shortly. The constitutive equation is the Generalized Newtonian Liquid (GNL), namely, $\boldsymbol{\tau} = \eta\dot{\boldsymbol{\gamma}}$, where $\boldsymbol{\tau}$ is the extra-stress tensor and $\dot{\boldsymbol{\gamma}} \equiv \operatorname{grad}\mathbf{v} + (\operatorname{grad}\mathbf{v})^{\mathrm{T}}$ the rate-of-deformation tensor, where $\mathbf{v}$ is the velocity vector. The viscosity function is given by the Herschel-Bulkley equation (Bird *et al.*, 1987):

$$\eta = \begin{cases} \frac{\tau_0}{\dot{\gamma}} + K\dot{\gamma}^{n-1}. & \text{if } \tau > \tau_0; \\ \infty, & \text{otherwise.} \end{cases} \tag{1}$$

In eq. (1), τ_0 is the yield stress, $\tau \equiv \sqrt{\frac{1}{2}\operatorname{tr}\boldsymbol{\tau}^2}$ is a measure of the magnitude of $\boldsymbol{\tau}$, $\dot{\gamma} \equiv \sqrt{\frac{1}{2}\operatorname{tr}\dot{\boldsymbol{\gamma}}^2}$ is a measure of the magnitude of $\dot{\boldsymbol{\gamma}}$, K is the consistency index, and n is the power-law exponent.

Two thermal boundary conditions are investigated for the inner wall: uniform heat flux (q_{w} = constant) and uniform temperature (T_{w} =constant). The outer wall was maintained insulated ($q_{\mathrm{w}} = 0$).

The dimensionless mass conservation equation is given by:

$$\frac{1}{r'}\frac{\partial}{\partial r'}\left(r'v'\right) + \frac{\partial}{\partial x'}\left(u'\right) = 0 \tag{2}$$

where $r' = r/\delta$ and $x' = x/\delta$ are the radial and axial dimensionless coordinates, $u' = u/\delta\dot{\gamma}_c$ is the dimensionless axial velocity, $v' = v/\delta\dot{\gamma}_c$ is the dimensionless radial velocity and $\delta = D_H/2 = R_o - R_i$ is the gap of the annular space.

Momentum conservation in axial and radial directions, respectively, is assured when the following dimensionless equations are satisfied:

$$\frac{1}{r'}\frac{\partial}{\partial r'}\left(r'v'u'\right) + \frac{\partial}{\partial x'}\left(u'u'\right) =$$

$$\frac{2\bar{u'}}{Re}\left[\frac{1}{r'}\frac{\partial}{\partial r'}\left(\eta' r'\frac{\partial v'_x}{\partial r'}\right) + \frac{\partial}{\partial x'}\left(\eta'\frac{\partial u'}{\partial x'}\right) + \frac{\partial \eta'}{\partial r'}\frac{\partial v'}{\partial x'} + \frac{\partial \eta'}{\partial x'}\frac{\partial u'}{\partial x'}\right] - \frac{\partial P'}{\partial x'} \tag{3}$$

$$\frac{1}{r'}\frac{\partial}{\partial r'}\left(r'v'v'\right) + \frac{\partial}{\partial x'}\left(v'u'\right) =$$

$$\frac{2\bar{u'}}{Re}\left[\frac{1}{r'}\frac{\partial}{\partial r'}\left(\eta' r'\frac{\partial v'}{\partial r'}\right) + \frac{\partial}{\partial x'}\left(\eta'\frac{\partial v'}{\partial x'}\right)\right] + \frac{2\bar{u'}}{Re}\left[\frac{\partial \eta'}{\partial r'}\frac{\partial v'}{\partial r'} + \frac{\partial \eta'}{\partial x'}\frac{\partial u'}{\partial r'} - \eta'\frac{v'}{r'^2}\right] - \frac{\partial P'}{\partial r'} \tag{4}$$

In the above expressions, $\eta' = \eta/\eta_c$ is the dimensionless viscosity and the dimensionless pressure P' is given by $P' = p/\tau_c$. The quantity τ_c is a characteristic shear stress, defined as $\tau_c \equiv -dp/dxR_o/2$. The characteristic viscosity is $\eta_c = \eta(\dot{\gamma}_c) = \tau_c/\dot{\gamma}_c$, where $\dot{\gamma}_c$ is the characteristic shear rate, given by $\dot{\gamma}_c = \left[(\tau_c - \tau_0)/K\right]^{1/n}$.

The dimensionless energy equation is given by:

$$\frac{1}{r'}\frac{\partial}{\partial r'}\left(r'v'\theta\right) + \frac{\partial}{\partial x'}\left(u'\theta\right) = \frac{2\bar{u'}}{Pe}\left[\frac{1}{r'}\frac{\partial}{\partial r'}\left(r'\frac{\partial \theta}{\partial r'}\right) + \frac{\partial}{\partial x'}\left(\frac{\partial \theta}{\partial x'}\right)\right] \tag{5}$$

In this equation $Pe \equiv \rho c_p \bar{u} D_H/k$ is the Péclet number, θ is the dimensionless temperature. For the case with uniform heat flux boundary condition, $\theta \equiv \frac{(T_{wi}-T)}{/q_{\mathrm{w}}D_H/k}$, where q_{w} is the heat flux at the inner wall and k is the heat conductivity. For this situation, the Nusselt number, $Nu = h_{wi}D_H/k$ at the inner wall is given by:

$$Nu = \frac{1}{\theta_b} \tag{6}$$

where θ_b is the dimensionless bulk temperature, given by:

$$\theta_b = \frac{\int \theta u' dA}{\int u' dA} \tag{7}$$

For cases with temperature boundary condition, $\theta \equiv \frac{T-T_{wi}}{T_b-T_{wi}}$, where T_{wi} is the temperature at the inner wall and $T_b = \frac{\int TudA}{\int udA}$ is the bulk temperature.

Modified bi-viscosity model

The viscosity function given by eq. (1) for Herschel-Bulkley materials is not easy to handle numerically. Then, an alternative model was proposed by Beverly and Tanner, in 1992, for Bingham materials, the so-called *bi-viscosity model*. This idea can be easily extended to Herschel-Bulkley materials, yielding the following expression for the viscosity function:

$$\eta' = \begin{cases} \frac{\tau_0'}{\dot\gamma'} + (1-\tau_0')\dot\gamma'^{n-1} & \text{if } \dot\gamma' > \dot\gamma'_{\text{small}} \\ \eta'_{\text{large}}, & \text{otherwise} \end{cases} \tag{8}$$

In the above equations, $\dot\gamma' \equiv \dot\gamma/\dot\gamma_c$ and $\tau_0' \equiv \tau_0/\tau_c$ are the dimensionless shear rate and yield stress, respectively. Beverly and Tanner (1992) recommend $\eta'_{\text{large}} = 1000$. Therefore, $\dot\gamma'_{\text{small}} = \tau_0'/[1000 - (1-\tau_0')\dot\gamma'^{n-1}_{\text{small}}] \simeq \tau_0'/1000$.

3. NUMERICAL SOLUTION

The governing equations presented above were discretized by the finite volume method described by Patankar (1980). Staggered velocity components are employed to avoid unrealistic pressure fields. The SIMPLE algorithm (Patankar, 1980) was used, in order to couple the pressure and velocity. The resulting algebraic system is solved by the TDMA line-by-line algorithm (Patankar, 1980) with the block correction algorithm (Settari and Aziz, 1973) to increase the convergence rate.

A non uniform mesh in axial direction with 102×62 points was used, concentrating more points near the inlet. Some mesh tests were performed in order to chose an adequate mesh. The error obtained for the developed value of the product $fRe^* = -8(dp/dx)\delta^2/\eta_c\bar{u}$ for a Newtonian fluid and $\delta/R_o = 0.5$ was compared with the ones found in the literature ($fRe^* = 95.2$) (Bejan, 1984) and an error of 0.5% was obtained. For the case with adiabatic inner wall and constant temperature outer wall, the error obtained for the Nusselt number Nu_{Dh}, relative to the value obtained in Incropera and Witt (1992) ($Nu_{Dh} = 4.43$), was equal to 1.6%.

The numerical solution was also compared with some limiting cases for non-Newtonian flows. The velocity field was compared with the exact solution obtained by Fredrickson e Bird (1958); the velocity field of a Herschel-Bulkley fluid with a ratio of radius tending to zero was compared to the exact solution for parallel plates; and the velocity field was compared with the experimental data obtained by Naimi *et al.* (1990). In all these cases, the agreement was quite good (Soares, 1999).

4. RESULTS

Results for some representative combinations of the governing parameters are now presented and discussed. Theoretical considerations show that there is a maximum value $\tau'_{0\text{crit}} = 1 - R_i/R_o$ (Fredrickson and Bird, 1958) for the dimensionless yield stress τ_0' beyond which no continuous solution is possible for the axial velocity field. This threshold value corresponds to the minimum pressure gradient needed to ensure that both wall shear stresses are larger than τ_0. The present paper is restricted to situations for which τ_0' is lower than this limiting value. Research for situations outside this range of $\tau'_{0\text{crit}}$ is underway.

Firstly, results for constant heat flux at the inner wall are presented. Dimensionless velocity and temperature profiles for different yield stress are shown in Figs. 1 and 2. It

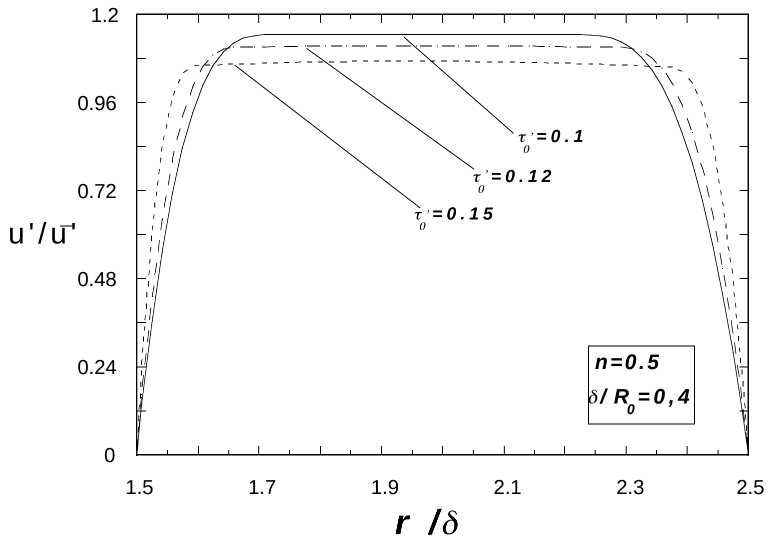

Figure 1: Velocity profile variation with τ_0'.

can be observed that higher yield stress cause the plug flow region to increase, as expected. However, the dimensionless temperature is almost invariant with τ_0' (not shown; see Soares, 1999).

Temperature variation along the annular space is shown in Fig. 2. It can be seen that temperature increases with the axial coordinate due to wall heating, as expected. It is worth observing in these figures that the curve slopes at both extremities of all curves are in accordance with the thermal boundary conditions.

Figs. 3–6 show the Nusselt number variation with flow and rheological parameters. It can be noted in Fig. 3 that the Nusselt number increases with the Péclet number, as expected. However, Nu is practically unaffected by the Reynolds number (not shown; see Soares, 1999). The effect of the rheological parameters (n and τ_0') can be analyzed with the aid of Figs. 4 and 5. It can be noted that higher values of Nusselt number are obtained as the fluid behavior departs from Newtonian (i.e., higher τ_0' and $n \to 0$). This is due to the velocity gradients near the wall, which increases as the mechanical behavior becomes more viscoplastic and/or pseudoplastic. The effect of the aspect ratio is shown in Fig. 6. In this figure, the Nusselt number is based in the outer tube diameter, which is held fixed. Increasing the annular space decreases the plug flow region. This behavior leads to lower velocity gradients at the walls and, consequently, lower heat transfer rates.

Only a few results for constant temperature at the inner wall are presented, due to space limitations. More details are found in Soares (1999). It can be said, however, that the qualitative results are quite similar to the ones obtained for the cases of constant heat flux. Figure 7 shows the dimensionless temperature profile, while Fig. 8 illustrates the dependence of the Nusselt number on the Péclet number. It is observed that the Nusselt number is less sensitive to the Péclet number than for the q_w = constant case.

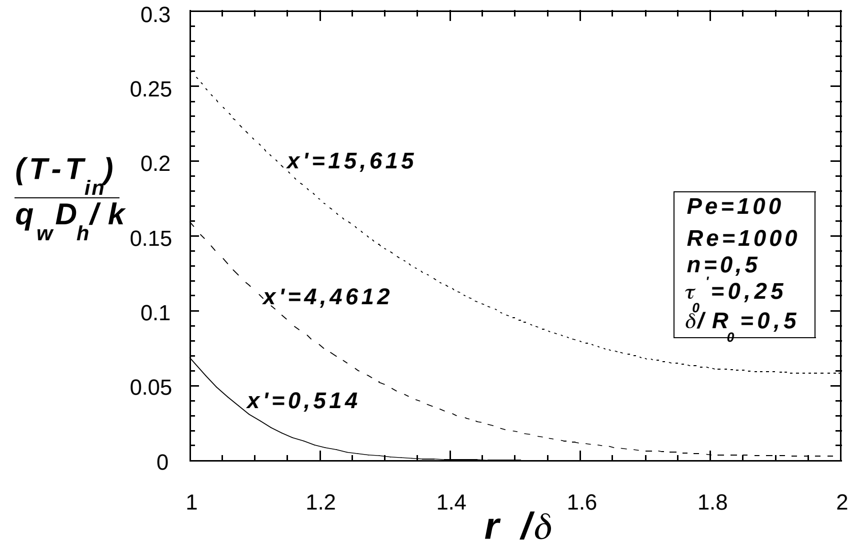

Figure 2: Dimensionless temperature profile for different axial positions, $q_w =$ constant.

5. CONCLUSIONS

This paper presented a study of the heat transfer problem for the flow of Herschel-Bulkley materials through annular spaces. Two temperature boundary conditions for the inner wall were analyzed: uniform heat flux and uniform temperature, while the outer wall was maintained insulated.

The governing equations were solved numerically via a finite-volume technique. Results are presented in the form of velocity and temperature profiles, and Nusselt number variation with some governing rheological and geometric parameters.

It was noted that Nusselt numbers are a little more sensitive to the parameters analyzed for the case of uniform heat flux.

It was also observed that the cases for which the velocity gradient at the wall is high yield higher Nusselt numbers. However, the Nusselt number variations are rather small for the range of rheological parameters analyzed. This result is quite important, because it indicates that the Newtonian values of Nusselt numbers can be used to estimate heat transfer for Herschel-Bulkley materials for the situation studied.

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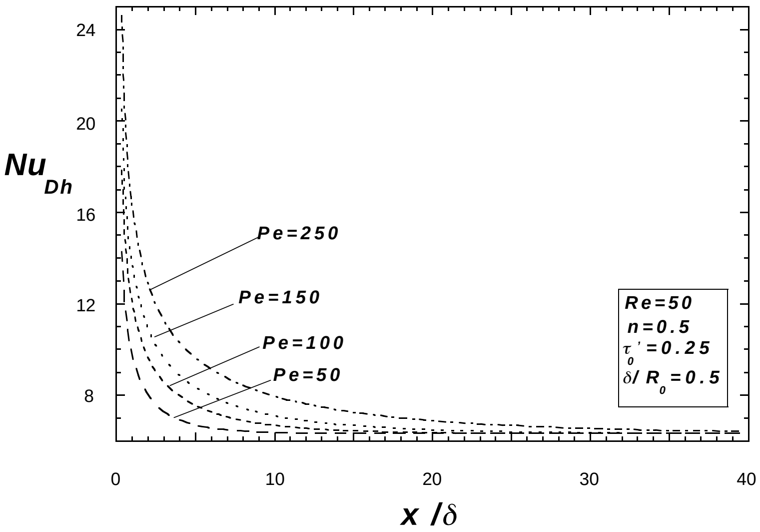

Figure 3: Nusselt number variation with Pe, $q_w =$ constant.

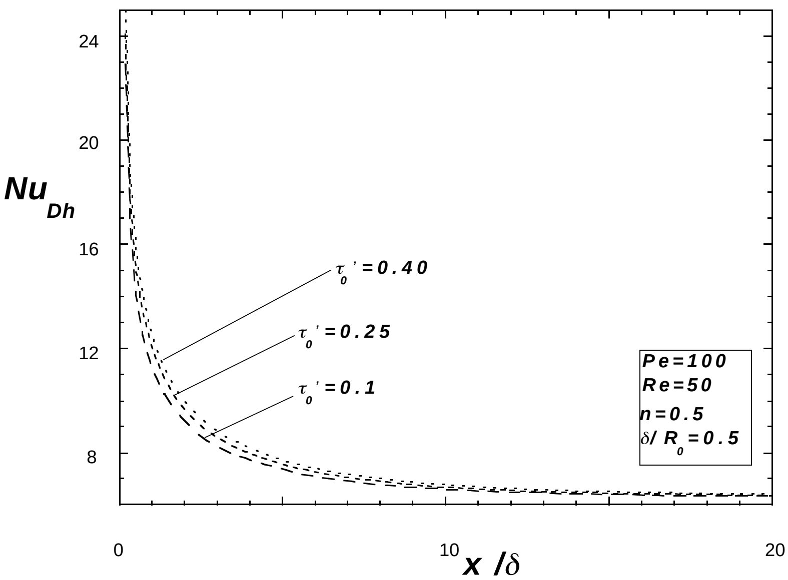

Figure 4: Nusselt number variation with τ_0', $q_w =$ constant.

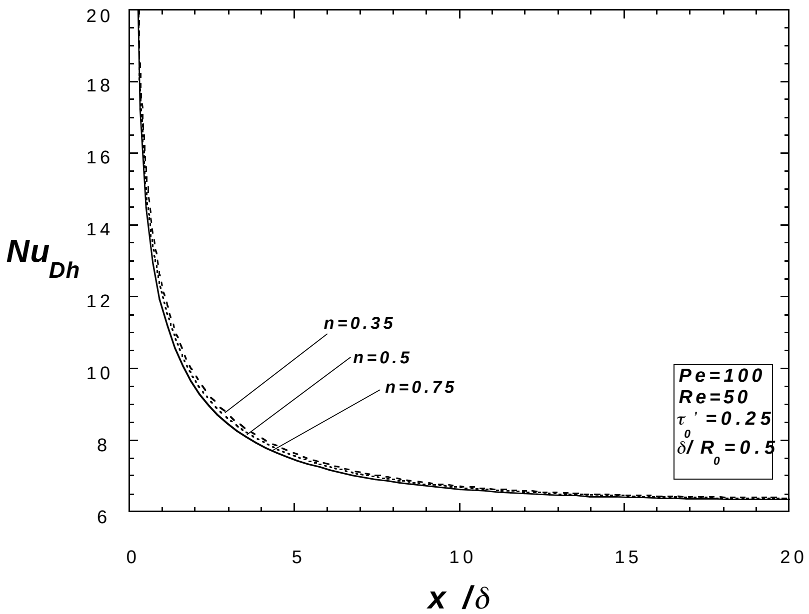

Figure 5: Nusselt number variation with n, $q_w =$ constant.

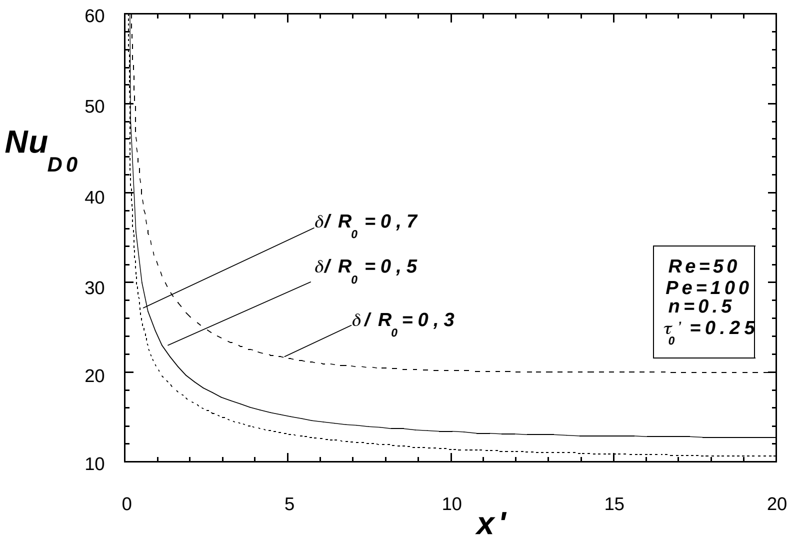

Figure 6: Nusselt number variation with δ/R_0, $q_w =$ constant.

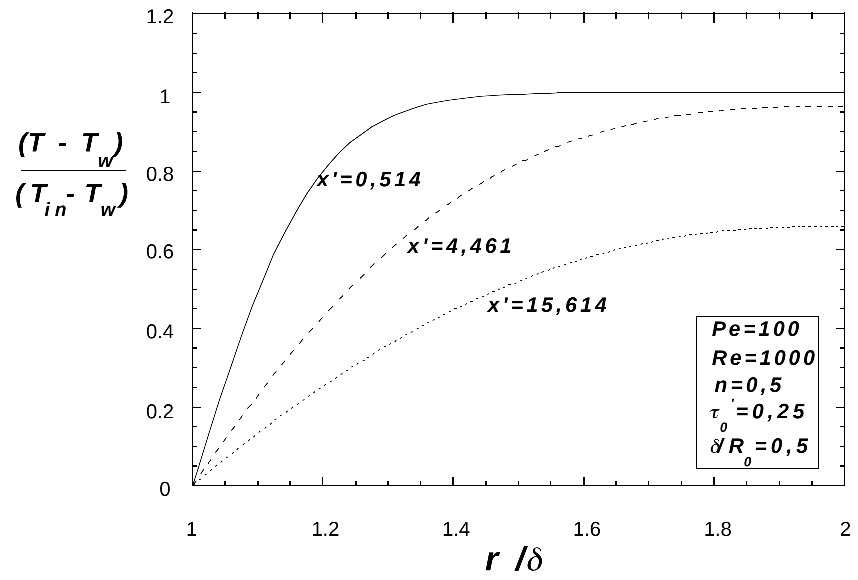

Figure 7: Dimensionless temperature profile for different axial positions, $T_w =$ constant.

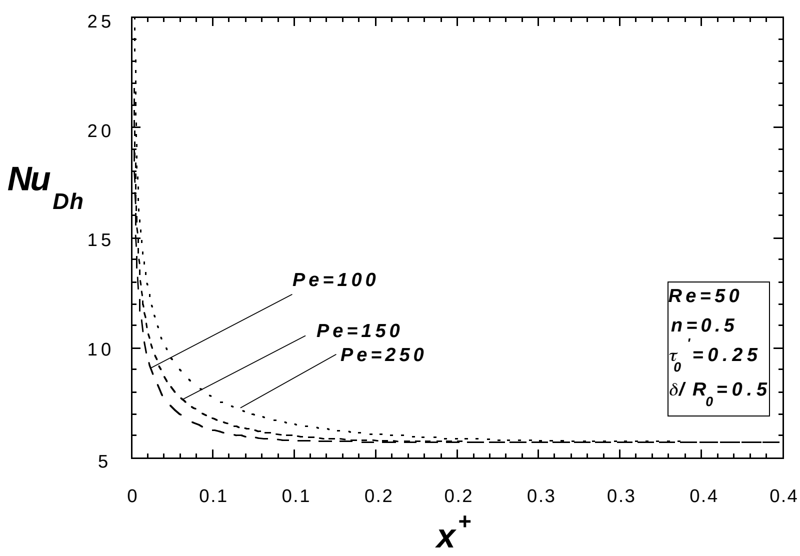

Figure 8: Nusselt number variation with Pe, $T_w =$ constant.